CURRENT TRANSFORMERS
An Analysis of Ratio and Phase Angle Error

Current transformers (C.T.'s) are basically of two general types - those that are used for overcurrent, relay protection applications and those that are used for metering applications. In protection applications one is concerned about the performance and errors of the C.T. when fault currents occur and under normal conditions the C.T. errors are of no concern. On the other hand metering applications errors are of concern under normal conditions and not of concern when faulted or abnormal conditions occur. In the first case, the C.T. is usually operating in magnetic saturation. The equivalent circuit for a C.T. for either application however is the same. The parameters of the circuits however, are different by design.

This is an approach to determine error involved when C.T.'s are to be used for metering applications. The error of particular concern is the phase shift which occurs across the C.T. under steady state conditions. The equivalent circuit is shown below

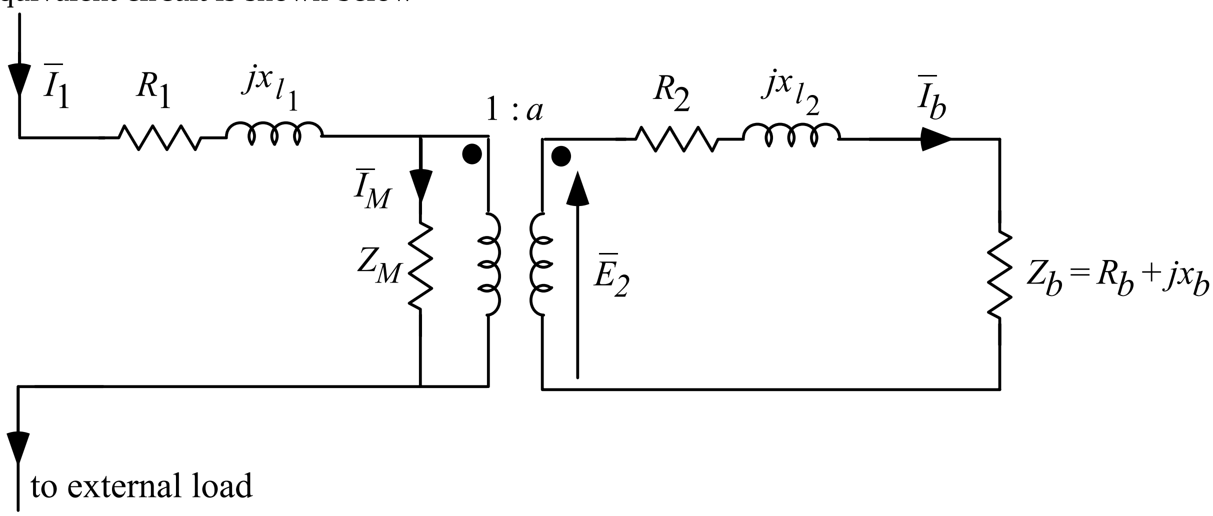

Figure 1

where R_1, R_2, and R_b are the resistances of the primary, secondary and the total burden on the transformer (i.e. lead resistance plus metering load) respectively. It is not necessary to be concerned about R_1 and leakage inductance x_{l_1} since I_1 is determined by the external line load. With this omission the circuit simplifies to

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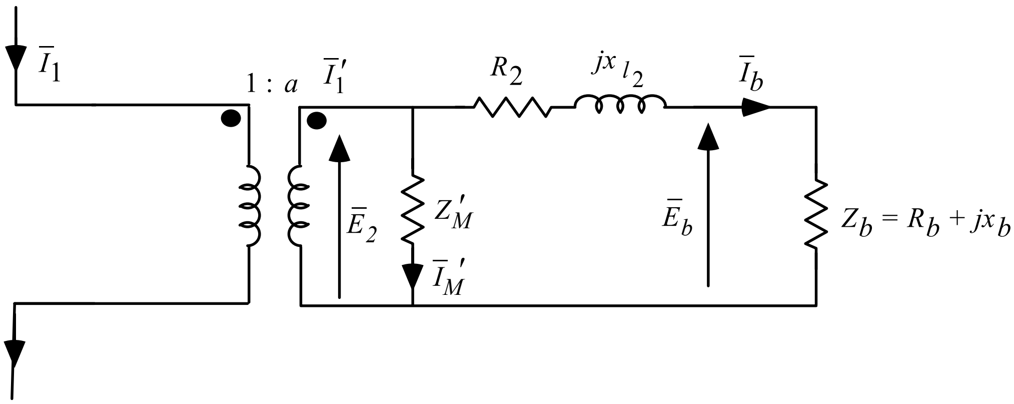
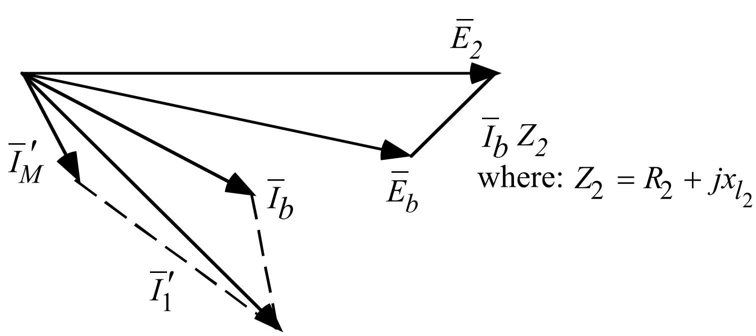


In the above circuits x_{l_1}, x_{l_2} and x_{l_b} are the inductive leakage reactances of the primary, secondary, and burden reactances respectively. Z_M is the magnetizing impedance the barred variables are indicative of phasor quantities. It is convenient to express primary quantities in secondary terms. Therefore current $\bar{I}_1 = \frac{\bar{I}_1'}{a}$ and the magnetizing Z_M becomes in secondary terms $Z_M' = \frac{Z_M}{a^2}$ and $I_M = \frac{I_M'}{a}$. where $a = \frac{N_p}{N_s}$.

It may be helpful to draw the phasor diagram of the secondary circuit as it now appears by taking $\bar{E}_2$ as reference and noting $\bar{I}_1' = \bar{I}_M' + \bar{I}_b$



The angle between $\bar{I}_1'$ and $\bar{I}_b$ is the C.T. current phase shift.

$$\bar{E}_2 = \bar{I}_b Z_2 + \bar{E}_b = \bar{I}_b\left(R_2 + jx_{x_{l_2}}\right) + \bar{I}_b\left(R_b + jx_b\right)$$

also $$\bar{E}_2 = \bar{I}'_M Z'_M$$

Note: Current phase shift between $\bar{I}'_1$ and $\bar{I}_b$ results because of the magnetizing current $\bar{I}'_M$. Normally the angle by which $\bar{I}'_M$ lags the voltage across it $(\bar{E}_2)$ is large.

Example Calculation

Assume a 250:5 C.T. ratio with a primary current of $\bar{I}_1 = 150\angle 0°$

Then:

$$\bar{I}'_1 = \frac{150\angle 0°}{50} = 3\angle 0° \quad \text{for} \quad a = \frac{5}{250}$$

By neglecting the primary leakage impedance as previously discussed and assuming the value of the secondary impedance as 0.03+j0.05 ohms, a total burden of 0.06 ohms and a magnetizing impedance $Z'_M = 7 + j10$ referred to the secondary the secondary voltage $\bar{E}_2$ is then

$$\bar{E}_2 = \bar{I}'_1 \frac{\left[R_2 + jx_{l_2} + R_b\right]\left[R'_M + jX'_M\right]}{\left[R_2 + jx_{l_2} + R_b + R'_M + jx'_M\right]} = \frac{\bar{I}'_1[0.03 + j0.05 + 0.06][7 + j10]}{0.03 + j0.05 + 0.06 + 7 + j10}$$

$$= \frac{\bar{I}'_1[0.09 + j0.05][7 + j10]}{7.09 + j10.05} = \frac{\bar{I}'_1[0.13 + j1.25]}{12.66\angle 56°}$$

$$= \bar{I}'_1\ [0.099\angle 31°]$$

$$\bar{E}_2 = 3\angle 0°[0.099\angle 31°] = 0.297\angle 31°$$

from which:

$$\bar{I}_M = \frac{0.297\angle 31°}{7 + j10} = 0.024\angle -24°$$

$$\bar{I}_b = \bar{I}'_1 - \bar{I}_M = 3\angle 0° - 0.024\angle -24° = 2.995\angle 0.03°$$

The phase shift between $\bar{I}_1'$ and $\bar{I}_b$ is $0.03°$. The phase shift is determined by the magnetizing current $\bar{I}_M'$.

If assume a new burden of 0.12 ohms for the same primary current of $\bar{I}_1 = 150\angle 0°$

$$\bar{E}_2 = \bar{I}_1' \frac{1.93\angle 73.5°}{12.33\angle 54.6°} = \bar{I}_1'(0.15\angle 18.9°)$$

$$\bar{E}_2 = 0.45\angle 18.9°$$

$$\bar{I}_M = \frac{0.45\angle 18.9°}{12.2\angle 55°} = 0.037\ \angle -36.1°$$

$$\bar{I}_b = 3 - \bar{I}_M = 2.971 + j0.022 = 2.97\angle 0.424°$$

Note phase shift in current has moved to $0.424°$.

In practical cases Z_M' does not remain constant because of magnetic saturation. Usually available are saturation characteristics as a plot of $\bar{E}_2$ as a function of the excitation current $\bar{I}_M'$. It is apparent from these non-linear curves that $Z_M' = \frac{\bar{E}_2}{\bar{I}_M'}$ is not constant as Z_M' is related to the slope of the excitation curve which is not constant and also a function of the C.T. ratio. For this reason one must include in the C.T. current phase shift calculations the non-linear saturation characteristic. To account for the non-linearity of Z_M' it is necessary to know the value of the secondary voltage $\bar{E}_2$ and $\bar{I}_M'$ for the particular value of the C.T. ratio taken off the excitation characteristic. The procedure here would be to plot the non-linear relationship on the excitation characteristic similar to what is done in non-linear electronic circuits the intersection of a "load line" defined by the non-linear characteristic is the operating point of the C.T. An example probably will be the best way to describe a solution to determine the phase shift which occurs.

Example calculation:

Assume a 250:5 C.T. ratio with $\bar{I}_1 = 200A$ then $\bar{I}_1 = 4A$ for $a = \frac{1}{50}$. Assume a constant burden $Z_b = R_B + jX_B = 3 + j0\Omega$. Let's assume $|Z_M'|$ is non-linear but the impedance angle of Z_M' remains constant at $70°$ that is $\tan^{-1}\frac{X_M}{R_M} = 70°$. The equivalent circuit is the same as before but Z_M' is non-linear

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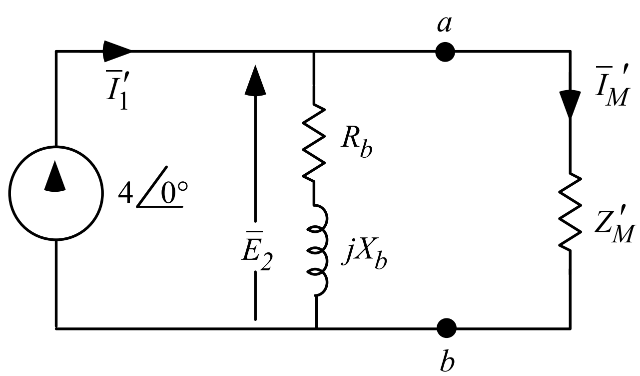
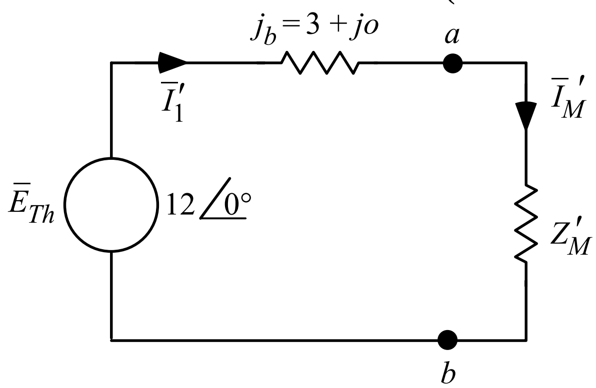
and not a constant magnitude. By assuming that the secondary leakage inductance is small compared to the burden impedance the calculations are easier with small error. however, if desired it can be included. By neglecting Z_2 we then have Z'_M in parallel with the constant burden Z_b. The equivalent circuit can be considered to be driven by an ideal current source equal to the transformed primary current $\bar{I}'_1$ in this case 4A. The circuit now can be rearranged as follows with $\bar{I}'_1$ as a phasor reference.



The Thevenin equivalent circuit for terminals *a* - *b* is:

$$\bar{E}_{Th} = \bar{I}_1\left[R_b + jX_b\right] = 4\angle 0°[3 + jo] = 12\angle 0°$$

(This is the Thevenin equivalent voltage)



The Thevenin impedance is $Z_b = 3 + jo$

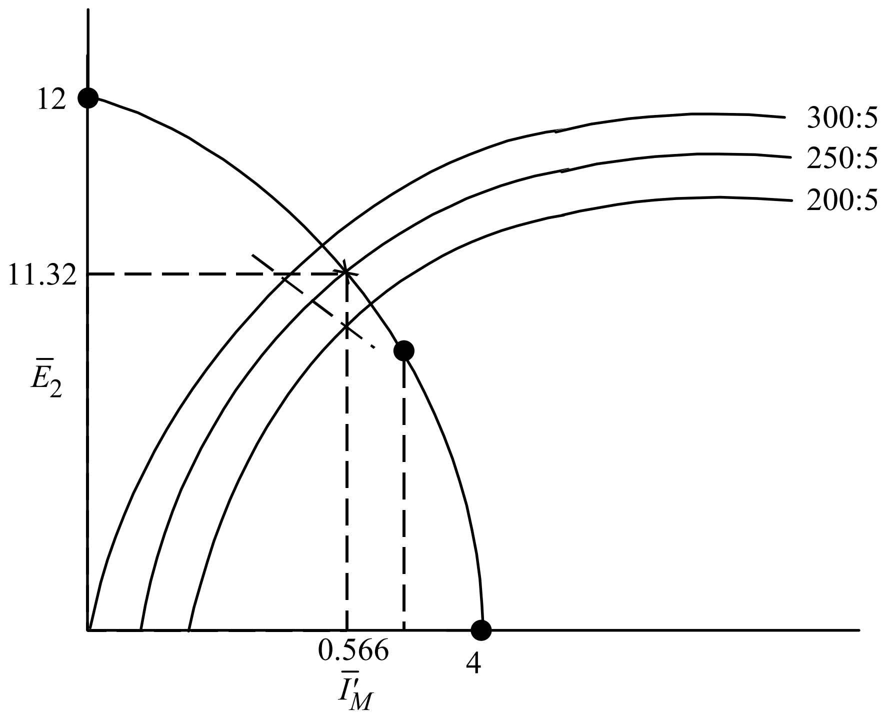
$$\bar{I}'_M = \frac{12\angle 0°}{Z_b + Z'_M} = \frac{12\angle 0°}{3 + j0 + |Z'_M|\angle 70°}$$

but $\quad \bar{E}_2 = \bar{I}'_M Z'_M$

These two equations describe $\bar{I}'_M$ and $\bar{E}_2$ which are the variables on the excitation characteristic. By now assuming magnitudes for $|Z'_M|$ we can plot the "load line" to obtain magnitudes of $\bar{E}_2$ and $\bar{I}'_M$.

$\bar{I}'_M$	$\lvert\bar{I}'_M\rvert$	$\lvert E_2\rvert$
0	$4\angle 0°$	0
∞	0	12
20	0.566	11.32
M	M	M

Let's assume the magnetizing characteristic is of typical form given and the load line intersection the 250:5 characteristic at $\bar{E}_2 = 11.32$ and $\bar{I}'_M = 0.566$.



Then we have

$$\bar{I}_1' = 4\angle 0^\circ$$

$$|Z_M'| = \bar{E}_2 / \bar{I}_M' = 11.32 / 0.566 = 20$$

$$I_M' = \frac{12\angle 0^\circ}{3 + 6.84 + j18.79} = 0.566\angle -62.3^\circ$$

$$\begin{aligned}
\bar{I}_b = \bar{I}_1' - \bar{I}_M' &= 4 - (0.566\angle -62\,3^\circ) \\
&= 4 - (0.263 - j\,0.5) \\
&= 3.74 + j0.5 \\
&= 3.77\angle 7.61^\circ
\end{aligned}$$

Therefore the phase shift between $\bar{I}_b'$ and $\bar{I}_b$ is 7.61°. This, of course, considers the saturation which occurs in the transformer.